Math 5, Homework 1

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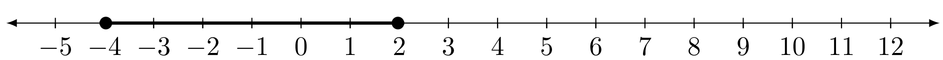
1.3 Q 1 Graph on a real line all points x such that:

$$(4x - 2 \leq 6 \text{ and } 3 - x \leq 7) \text{ or } (3x > 18 \text{ and } 2x - 9 \geq 11)$$

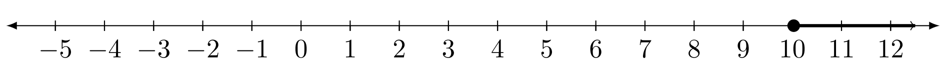
The four inequalities above can be written more cleanly as the following four inequalities:

$$(x \leq 2 \text{ and } x \geq -4) \text{ or } (x > 6 \text{ and } x \geq 10)$$

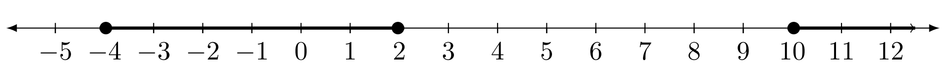
The word "and" between two inequalities indicates that we need them to simultaneously be true. This corresponds to taking the intersection of their individual solution sets. So the first pair of inequalities are true for all x such that $-4 \leq x \leq 2$. On a number line, this looks like:

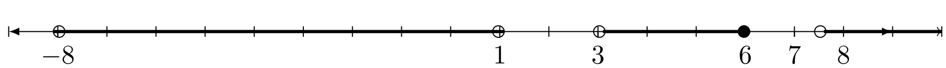


Similarly for the other pair of inequalities, the intersection of their solution sets are all x such that $x \geq 10$.



Now the word "or" between two inequalities indicates that we need either one of them to be true, which corresponds to taking the union of their solution sets. So all the points x that satisfy those inequalities and conditions, when graphed on a number line, would look like





1.4 Q 2 The force F between two bodies of mass m_1 kilograms and m_2 kilograms whose centers of mass are a distance r meters apart is given by the formula

$$F = \frac{Gm_1m_2}{r^2},$$

where G is the gravitational constant of the universe. What are the units of the gravitational constant G?

Since we want to figure out what the units of G are, first we should rewrite the given formula with G isolated:

$$F = \frac{Gm_1m_2}{r^2} \quad \Longrightarrow \quad G = \frac{Fr^2}{m_1m_2}$$

Since this is an equality, the units on the left-hand side (so the units of G) will be the same as the units on the right-hand side. We recall that since $F = ma$ the units of force are $\mathrm{kg}(\mathrm{m}/\mathrm{s}^2)$. So the units of the right-hand side must be

$$\frac{\mathrm{kg}\,\frac{\mathrm{m}}{\mathrm{s}^2}\,\mathrm{m}^2}{\mathrm{kg}\,\mathrm{kg}},$$

which we can write more cleanly as

$$\frac{\mathrm{m}^3}{\mathrm{kg}\,\mathrm{s}^2}.$$

So these are the units of G.

1.5 Q 2 The weight of paint is proportional to its volume. Paint is always applied at the same thickness. It takes 10 ounces of paint to paint a ball that has a volume of 3 cubic feet. How many ounces of paint does it take to paint a ball that has a volume of 24 cubic feet?

Say that you have two balls, one with a volume of 3 cubic feet, and the other with a volume of 24 cubic feet. We'll say that the large ball is k times bigger than the small ball, where k is the 1-dimensional scaling of the balls. Since we are going to think about surface area as a 2-dimensional measurement of the balls, the surface area will scale by a factor of k^2, and similarly since volume is a 3-dimensional measurement of the balls, the volume will scale by a factor of k^3.

We are given the volumes of the two balls, and see that the volume scaled by a factor of 8. In terms of our 1-dimensional scaling factor k, this means that $k^3 = 8$, so the 1-dimensional measurements of the ball scaled by a factor of $\sqrt[3]{8}$ which is 2.

Now to find the surface area of the larger ball (which represents the amount of paint we would need to paint the larger ball), we just scale the surface area of the smaller ball by a factor of k^2. So it must take $(10\mathrm{oz})(2^2) = 40\mathrm{oz}$ of paint to paint the larger ball.